Intermediate inflation in Gauss-Bonnet braneworld

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Abstract

In this article we study an intermediate inflationary universe models using the Gauss-Bonnet brane. General conditions required for these models to be realizable are derived and discussed. We use recent astronomical observations to constraint the parameters appearing in the model.

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I. INTRODUCTION

It is well known that one of the most exciting ideas of contemporary physics is to explain the origin of the observed structures in our universe. It is believed that Inflation [1] can provide an elegant mechanism to explain the large-scale structure, as a result of quantum fluctuations in the early expanding universe, predicting that small density perturbations are likely to be generated in the very early universe with a nearly scale-free spectrum [2]. This prediction has been supported by early observational data, specifically in the detection of temperature fluctuations in the cosmic microwave background (CMB) by the COBE satellite [3]. The scheme of inflation [4] (see [5] for a review) is based on the idea that at early times there was a phase in which the universe evolved through accelerated expansion in a short period of time at high energy scales. During this phase, the universe was dominated by a potential $V(\phi)$ of a scalar field ϕ (inflaton).

In the context of inflation we have the particular scenario of "intermediate inflation", in which the scale factor evolves as $a(t) = \exp(At^f)$. Therefore, the expansion of the universe is slower than standard de Sitter inflation ($a(t) = \exp(Ht)$), but faster than power law inflation ($a(t) = t^p; p > 1$). The intermediate inflationary model was introduced as an exact solution for a particular scalar field potential of the type $V(\phi) \propto \phi^{-4(f^{-1}-1)}$, where f is a free parameter[6]. Recently, a tachyon field in intermediate inflation was considered in [7], and a warm-intermediate inflationary universe model was studied en Ref.[8] (see also Ref.[9]).

The motivation to study intermediate inflationary model becomes from string/M theory (for a review see Refs.[10]). This theory suggests that in order to have a ghost-free action high order curvature invariant corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term[11]. GB terms arise naturally as the leading order of the expansion to the low-energy string effective action, where is the inverse string tension[12]. This kind of theory has been applied to possible resolution of the initial singularity problem[13], to the study of Black- Hole solutions[14], accelerated cosmological solutions[15], among others (see Refs.[16–22]). In particular, very recently, it has been found that for a dark energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form $a = \exp At^f$ [23], where the universe starts evolving with a decelerated exponential expansion. Here, the constant A becomes given by $A = \frac{2}{\kappa n}$ and $f = \frac{1}{2}$, with $\kappa^2 = 8\pi G$ and n is a constant. Also, much attention has been focused on the Randall

Sundrum (RS) scenario, where our observable four-dimensional universe is modelled as a domain wall embedded in a higher-dimensional bulk space [24]. These kind of models can be obtained from superstring theory [25, 26]. For a comprehensible review on RS cosmology, see Refs. [27–29]. In this way, the idea that inflation , or specifically, intermediate inflation, comes from an effective theory at low dimension of a more fundamental string theory is in itself very appealing. Thus, in brane universe models the effective theories that emerge from string/M theory lead to a Friedmann Equation which is proportional to the square energy density.

When the five dimensional Einstein-GB equations are projected on to the brane, a complicated Hubble equation is obtained [30–32]. Interestingly enough, this modified Friedmann equation reduces to a very simple equation $H^2 \propto \rho^q$ with $q = 1, 2, 2/3$ corresponding to General Relativity (GR), RS and GB regimes, respectively. This situation motivated the "patch cosmology" as a useful approach to study braneworld scenarios [33]. This scheme makes use of a nonstandard Friedmann equation of the form $H^2 = \beta_q^2 \rho^q$. Despite all the shortcomings of this approximate treatment of extra-dimensional physics, it gives several important first-impact information. Recently, a closed inflationary universe in patch cosmology was considered in [34], and a tachyonic universes in patch cosmologies with $\Omega > 1$ was studied in Ref.[35].

The purpose of the present work is to study intermediate inflationary universe models, where the matter content is confined to a four dimensional brane which is embedded in a five dimensional bulk where a GB contribution is considered. We study these models using the approach of patch cosmology. On the other hand, a comprehensive study in the present work reveals that, intermediate inflation provides the possibility of density perturbation and gravitational wave spectra which differ from the usual inflationary prediction of a nearly flat spectrum with negligible gravitational waves. Furthermore, in the present model the tensor-to-scalar ratio r is scale-dependent, and we have shown that a good fit to the WMAP5 observations.

The outline of the Letter is as follows. The next section we briefly review the cosmological equations in the GB brane world and present the patch cosmological equations for this model. In Sect. III presents a short review of the intermediate inflation in GB brane. In Sect. IV the cosmological perturbations are investigated. Finally, in Sect. V we summarize our finding.

II. COSMOLOGICAL EQUATIONS IN GAUSS-BONNET BRANE

We start with the five-dimensional bulk action for the GB braneworld:

$$S = \frac{1}{2\kappa_5^2}\int_{bulk} d^5x\sqrt{-g_5}\left\{R - 2\Lambda_5 + \alpha\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\nu\mu} + R^2\right)\right\} + \int_{brane} d^4x\sqrt{-g_4}\left(\mathcal{L}_{matter} - \sigma\right), \quad (1)$$

where $\Lambda_5 = -3\mu^2\left(2 - 4\alpha\mu^2\right)$ is the cosmological constant in five dimensions, with the AdS_5 energy scale μ, α is the GB coupling constant, $\kappa_5 = 8\pi/m_5$ is the five dimensional gravitational coupling constant and σ is the brane tension. $\mathcal{L}_{matter}$ is the matter lagrangian for the inflaton field on the brane. We will consider the case that a perfect fluid matter source with density ρ is confined to the brane.

A Friedmann-Robertson-Walker (FRW) brane in an AdS$_5$ bulk is a solution to the field and junction equations (see Refs.[30–32]). The modified Friedmann on the brane can be written as

$$H^2 = \frac{1}{4\alpha}\left[(1 - 4\alpha\mu^2)\cosh\left(\frac{2\chi}{3}\right) - 1\right], \quad (2)$$

$$\kappa_5^2(\rho + \sigma) = \left[\frac{2(1 - 4\alpha\mu^2)^3}{\alpha}\right]^{1/2}\sinh\chi\,, \quad (3)$$

where χ represents a dimensionless measure of the energy density ρ. In this work we will assume that the matter fields are restricted to a lower dimensional hypersurface (brane) and that gravity exists throughout the space-time (brane and bulk) as a dynamical theory of geometry. Also, for 4D homogeneous and isotropic Friedmann cosmology, an extended version of Birkhoffs theorem tells us that if the bulk space-time is AdS, it implies that the effect of the Weyl tensor (known as dark radiation) does not appear in the modified Friedmann equation. On the other hand, the brane Friedmann equation for the general, where the bulk spacetime may be interpreted as a charged black hole was studied in Refs.[36–38].

The modified Friedmann equation (2), together with Eq. (3), shows that there is a characteristic Gauss-Bonnet energy scale[39]

$$m_{GB} = \left[\frac{2(1 - 4\alpha\mu^2)^3}{\alpha\kappa_5^4}\right]^{1/8}, \quad (4)$$

such that the GB high energy regime ($\chi \gg 1$) occurs if $\rho + \sigma \gg m_{GB}^4$. Expanding Eq. (2) in χ and using (3), we find in the full theory three regimes for the dynamical history of the brane universe [30–32]:

$$\rho \gg m_{GB}^4 \;\Rightarrow\; H^2 \approx \left[\frac{\kappa_5^2}{16\alpha}\,\rho\right]^{2/3} \qquad (GB), \tag{5}$$

$$m_{GB} \gg \rho \gg \sigma \Rightarrow\; H^2 \approx \frac{\kappa_4^2}{6\sigma}\,\rho^2 \qquad (RS), \tag{6}$$

$$\rho \ll \sigma \;\Rightarrow\; H^2 \approx \frac{\kappa_4^2}{3}\,\rho \qquad (GR). \tag{7}$$

Clearly Eqs. (5), (6) and (7) are much simpler than the full Eq (2) and in a practical case one of the three energy regimes will be assumed. Therefore, patch cosmology can be useful to describe the universe in a region of time and energy in which [33]

$$H^2 = \beta_q^2 \rho^q, \tag{8}$$

where $H = \dot{a}/a$ is the Hubble parameter and q is a patch parameter that describes a particular cosmological model under consideration. The choice $q = 1$ corresponds to the standard General Relativity with $\beta_1^2 = 8\pi/3m_p^2$, where m_p is the four dimensional Planck mass. If we take $q = 2$, we obtain the high energy limit of the brane world cosmology, in which $\beta_2^2 = 4\pi/3\sigma m_p^2$. Finally, for $q = 2/3$, we have the GB brane world cosmology, with $\beta_{2/3}^2 = G_5/16\zeta$, where G_5 is the $5D$ gravitational coupling constant and $\zeta = 1/8g_s$ is the GB coupling (g_s is the string energy scale). The parameter q, which describes the effective degrees of freedom from gravity, can take a value in a non-standard set because of the introduction of non-perturbative stringy effects. Here, we mentioned some possibilities, for instance, in Ref.[21] it was found that an appropriate region to a patch parameter q is given by $1/2 = q < \infty$. On the other hand, from Cardassian cosmology it is possible to obtain a Friedmann equation similar (8) as a consequence of embedding our observable universe as a 3+1 dimensional brane in extra dimensions. In fact, in Ref.[40] a modified FRW equation was obtained in our observable brane with $H^2 \propto \rho^n$ for any n.

On the other hand, we neglect any contribution from both the Weyl tensor and the brane-bulk exchange, assuming there is some confinement mechanism for a perfect fluid. Thus, the energy conservation equation on the brane follows directly from the Gauss-Codazzi equations. For a perfect fluid matter source it is reduced to the familiar form, $\dot{\rho}+3H\,(\rho + P) = 0$,

where ρ and P represent the energy and pressure densities, respectively. The dot denotes derivative with respect to the cosmological time t.

We consider that the matter content of the universe is a homogeneous inflaton field $\phi(t)$ with potential $V(\phi)$. Then the energy density and pressure are given by $\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$ and $P = \frac{\dot{\phi}^2}{2} - V(\phi)$, respectively. In this way, the equation of motion of the rolling scalar field becomes

$$\ddot{\phi} + 3\, H\, \dot{\phi} + V'(\phi) = 0\ , \tag{9}$$

here, for convenience we will use units in which $c = \hbar = 1$ and $V'(\phi) = \partial V(\phi)/\partial\phi$.

III. INTERMEDIATE INFLATION IN GAUSS BONNET BRANE

In this section exact solutions can be found for intermediate inflationary universes models where the scale factor, $a(t)$, expands as follows

$$a(t) = \exp(At^f). \tag{10}$$

Here f is a constant parameter with range $0 < f < 1$, and A is a positive constant.

From equations (8), (9), and using (10), we obtain

$$\dot{\phi}^2 = -\frac{2\dot{H}H^{2\alpha_0}}{3q\beta_q^{2(\alpha_0+1)}} = \frac{2(Af)^{2\alpha_0+1}(1-f)}{3q\beta_q^{2(\alpha_0+1)}}\ t^{2\alpha_1}, \tag{11}$$

and the effective potential as a function of the cosmological times becomes

$$V(t) = \left[\frac{(Af)^2\, t^{2(f-1)}}{\beta_q^2}\right]^{\frac{1}{q}} - \frac{(Af)^{2\alpha_0+1}\,(1-f)}{3q\beta_q^{2(\alpha_0+1)}} t^{2\alpha_1}, \tag{12}$$

where

$$\alpha_0 = \frac{1-q}{q}, \quad \text{and} \quad \alpha_1 = f(\alpha_0 + \frac{1}{2}) - (\alpha_0 + 1),$$

are constant parameters, respectively.

The solution for the scalar field $\phi(t)$ can be found from Eq.(11)

$$(\phi - \phi_0) = A^{\alpha_0+\frac{1}{2}}\alpha_2 t^{\alpha_1+1}, \tag{13}$$

where $\phi(t=0) = \phi_0$. Here, the parameter α_2 is defined by

$$\alpha_2 = \left[\frac{2f^{2\alpha_0+1}(1-f)}{3q\beta_q^{2(\alpha_0+1)}(\alpha_1+1)^2}\right]^{\frac{1}{2}}.$$

An exact solution of Eqs. (12) and (13) of the form of Eq.(10) exists with

$$V(\phi) = A^{\frac{2\alpha_3}{q}}\alpha_4(\phi-\phi_0)^{\frac{2(f-1)}{q(\alpha_1+1)}} - \frac{A^{\frac{2(\alpha_0+\frac{1}{2})}{\alpha_1+1}}}{2}\alpha_5(\phi-\phi_0)^{\frac{2\alpha_1}{\alpha_1+1}}, \tag{14}$$

where

$$\alpha_3 = 1 + \frac{(\alpha_0+\frac{1}{2})(1-f)}{(\alpha_1+1)}, \quad \alpha_4 = \left[\frac{f^2\alpha_2^{\frac{2(1-f)}{\alpha_1+1}}}{\beta_q^2}\right]^{\frac{1}{q}}, \text{ and } \quad \alpha_5 = \alpha_2^{\frac{2}{\alpha_1+1}}(\alpha_1+1)^2.$$

The Hubble parameter as a function of the inflaton field ϕ becomes

$$H(\phi) = A^{\alpha_3} f \alpha_2^{\frac{1-f}{\alpha_1+1}}(\phi-\phi_0)^{\frac{f-1}{\alpha_1+1}}. \tag{15}$$

Assuming the set of slow-roll conditions, $\frac{\dot{\phi}^2}{2} \ll V(\phi)$ and $\ddot{\phi} \ll 3H\dot{\phi}$, the potential given by Eq.(14) reduces to

$$V(\phi) = A^{\frac{2\alpha_3}{q}}\alpha_4(\phi-\phi_0)^{\frac{2(f-1)}{q(\alpha_1+1)}}. \tag{16}$$

Here, the first term of the effective potential given by Eq. (14) dominates at large values of $(\phi-\phi_0)$. Note that, the solutions for $\phi(t)$ and $H(\phi)$, corresponding to this potential are identical to those obtained when the exact potential, Eq. (12), is used.

We should note that in the GR regime, i.e., $q=1$ the scalar potential becomes $V(\phi) \propto \phi^{-4(1-f)/f}$, in the RS regime i.e., $q=2$, the potential is $V(\phi) \propto \phi^{-2(1-f)}$, and finally in the GB regimen $q=2/3$, $V(\phi) \propto \phi^{-3(1-f)/(f-1/2)}$. Without loss of generality ϕ_0 can be taken to be zero. Note that the potentials which are asymptotically of inverse power-law type are commonly used in quintessence models [41], but it also establishes viable inflationary solutions. These potentials also arises from the scalar-tensor gravity theories[42].

Introducing the Hubble slow-roll parameters (ϵ_1, η_η) and potential slow-roll parameters (ϵ_1^q, η_n^q), see Ref.[19], we write

$$\epsilon_1 = -\frac{\dot{H}}{H^2} \approx \epsilon_1^q = \frac{qV'^2}{6\beta_q^2 V^{q+1}} = A^{2\alpha_0\alpha_3}\alpha_6\phi^{-2\gamma}, \tag{17}$$

and

$$\eta_n = -\frac{1}{H^n\dot{\phi}}\frac{d^{n+1}\phi}{dt^{n+1}} \approx \eta_n^q, \tag{18}$$

where

$$\eta_1^q = \frac{1}{3\beta_q^2}\left[\frac{V''}{V^q} - \frac{qV'^2}{2V^{q+1}}\right] = A^{2\alpha_0\alpha_3}\alpha_7\phi^{-2\gamma},$$

$$\eta_2^q = \frac{-1}{(3\beta_q^2)^2}\left[\frac{V'V''}{V^{2q}} + \frac{(V'')^2}{V^{2q}} - \frac{5qV''(V')^2}{V^{2q+1}} + \frac{q(q+2)(V')^4}{2\,V^{2(q+1)}}\right]$$
$$= \left[\frac{-a^2\, B^{2(1-q)}\,\phi^{2a(1-q)-4}}{(3\beta_q^2)^2}\right]\,[1 + a^2(1+q[q-8]/2) - \phi + a(\phi + 5q - 2)].$$

Here, the parameters γ, α_6, α_7, a and B are

$$\gamma = \frac{\alpha_0(1-f)}{(\alpha_1+1)} + 1, \quad \alpha_6 = \frac{q\alpha_4^{1-q}}{6\beta_q^2}\left[\frac{2(1-f)}{q(\alpha_1+1)}\right]^2,$$

$$\alpha_7 = \frac{2(1-f)\alpha_4^{1-q}}{q(\alpha_1+1)3\beta_q^2}\left[\frac{(1-f)}{(\alpha_1+1)}\left(\frac{q}{2}-1\right)+1\right], \quad a = \frac{2(f-1)}{q(\alpha_1+1)}, \text{ and } B = \alpha_4\, A^{2\alpha_3/q},$$

respectively.

Note that, the ratio between η_1^q and ϵ_1^q becomes

$$\frac{\eta_1^q}{\epsilon_1^q} = \frac{2}{q} + \frac{\alpha_1+1}{1-f} - 1, \tag{19}$$

and η_1^q reaches unity before ϵ_1^q does. Therefore, we may establish that the end of inflation is governed by the condition $\eta_1^q = 1$ in place of $\epsilon_1^q = 1$. From this, we get for the scalar field ϕ at the end of inflation, becomes

$$\phi_{end} = \left(A^{2\alpha_0\alpha_3}\alpha_7\right)^{\frac{1}{2\gamma}}. \tag{20}$$

On the other hand, the number of e-folds at the end of inflation using Eqs. (8), (9) and (16) under the set of slow-roll conditions, is given by

$$N = \int_{t_*}^{t_{end}} H dt = 3\beta_q^2\int_{\phi_{end}}^{\phi_*}\frac{V^q}{V'}d\phi = A^{-2\alpha_0\alpha_3}\alpha_8\left\{\phi_{end}^{2\gamma} - \phi_*^{2\gamma}\right\}, \tag{21}$$

where

$$\alpha_8 = \frac{3\beta_q^2\alpha_4^{q-1}q(\alpha_1+1)}{4(1-f)\gamma}.$$

The subscripts $*$ and *end* are used to denote the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

IV. PERTURBATION SPECTRAL FROM INTERMEDIATE INFLATION IN PATCH COSMOLOGICAL MODELS

In this section we will study the scalar and tensor perturbations for our model. It has long been recognized that inflation gives rise to a spectrum of scalar perturbations close to the

scale-invariant Harrison-Zel'dovich. For a scalar field the amplitude of scalar perturbations generated during inflation for a flat space is approximately [19]

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_{k=k_*} = A^{2\xi}\alpha_9\phi_*^{2\sigma}, \tag{22}$$

where

$$\alpha_9 = \frac{f^4\alpha_2^{\frac{4(1-f)}{\alpha_1+1}}}{4\pi^2\alpha_5}, \qquad \xi = 2\alpha_3 - \frac{(\alpha_0+\frac{1}{2})}{(\alpha_1+1)}, \quad \text{and} \quad \sigma = \frac{[2(f-1)-\alpha_1]}{(\alpha_1+1)},$$

respectively. Here we have used Eqs. (11), and (15). The quantity k_*, is refereed to $k = Ha$, the value when the universe scales crosses the Hubble Horizon during inflation.

From Eqs. (20), (21) and (22), we obtained a constraint for the parameter A given by

$$A = \left\{\frac{\mathcal{P}_{\mathcal{R}}}{\alpha_9}\left[\frac{N}{\alpha_8} - \alpha_7\right]^{-\frac{\sigma}{\gamma}}\right\}^{\frac{1}{2\left[\xi+\alpha_0\alpha_3\frac{\sigma}{\gamma}\right]}}. \tag{23}$$

In this way, we can obtain the value of A for a given values of f and β_q^2 parameters when number of e-folds N, and the power spectrum of the curvature perturbations $\mathcal{P}_{\mathcal{R}}$ is given. Now we consider the special case in which $f = 2/3$. In this special case we obtained that in GR ($q = 1$) we get $A \simeq 0.0019 m_p^{2/3}$. In RS ($q = 2$) for the value of $\beta_2^2 = 10^{-13} m_p^{-6}$, we have $A \simeq 0.0014 m_p^{2/3}$, and in the GB regime ($q = 2/3$) for $\beta_{2/3}^2 = 10^{-3} m_p^{-2/3}$, we get $A \simeq 0.0046 m_p^{2/3}$. Here we have taken $N = 60$ and $\mathcal{P}_{\mathcal{R}} \simeq 2.4 \times 10^{-9}$.

Note that the general expression for the amplitude of scalar perturbations in GB brane world is given by[39]

$$\mathcal{P}_{\mathcal{R}} = \left[\frac{\kappa_4^6\, V^3}{6\pi^2\, V'^2}\right]\, G_\beta^2(H/\mu)_{\;k=k_*}, \tag{24}$$

where the term in square brackets is the standard scalar perturbation, and the GB brane world correction is given by

$$G_\beta^2(x) = \left[\frac{3\,(1+\beta)\,x^2}{2(3-\beta+2\beta\,x^2)\sqrt{1+x^2}+2(\beta-3)}\right]^3,$$

where $x \equiv H\mu$ is a dimensionless measure of energy scale, and $\beta = 4\alpha\mu$. The RS amplification factor is recovered when $\beta = 0$[43].

We also consider the q-spectral index n_s^q, which is related to the power spectrum of density perturbations $\mathcal{P}_{\mathcal{R}}$. For modes with a wavelength much larger than the horizon ($k \ll aH$), where k is the comoving wave number. The scalar q-spectral index is given by $n_s^q = 1 + d\ln\mathcal{P}_{\mathcal{R}}/d\ln k$, see Ref.[22], and in our case becomes

$$n_s^q = 1 - 4\epsilon_1^q + 2\eta_1^q = 1 - \frac{2A^{2\alpha_0\alpha_3}\alpha_4^{1-q}}{3\beta_q^2}\left[\frac{2(1-f)}{q(\alpha_1+1)}\right]\left[\frac{(1-f)}{(\alpha_1+1)}\left(3-\frac{2}{q}\right)-1\right]\phi^{-2\gamma}. \quad (25)$$

In order to confront these models with observations, we need to consider the q-tensor-scalar ratio $r_q = 16\, A_{T,q}^2/A_{S,q}^2$, where the q-scalar amplitude is normalized by $A_{S,q}^2 = 4\mathcal{P}_{\mathcal{R}}/25$. Here, the tensor amplitude is given by

$$A_{T,q}^2 = A_{T,GR}^2\, F_\beta^2(H/\mu), \quad (26)$$

where $A_{T,GR}^2$ is the standard amplitude in GR i.e., $A_{T,GR} = 24\beta_1^2\,(H/2\pi)^2$, and the function F_β contains the information about the GB term [39]

$$F_\beta^{-2} = \sqrt{1+x^2} - \left(\frac{1-\beta}{1+\beta}\right) x^2 \sinh^{-1}\left(\frac{1}{x}\right) \qquad (x \equiv \frac{H}{\mu}).$$

Following, Ref.[33] we approximate the function $F_\beta^2 \approx F_q^2$, where for the GR regime $F_{q=1}^2 \approx F_\beta^2(H/\mu \ll 1) = 1$, for the RS regime $F_{q=2}^2 \approx F_{\beta=0}^2(H/\mu \gg 1) = 3H/(2\mu)$, and finally for the GB regime $F_{q=2/3}^2 \approx F_\beta^2(H/\mu \gg 1) = (1+\beta)H/(2\beta\mu)$. The tensor amplitude up to leading-order is given by

$$A_{T,q}^2 = \frac{3q\beta_q^{2-2(1-q^{-1})}}{(5\pi)^2}\frac{H^{2+2(1-q^{-1})}}{2\zeta_q}, \quad (27)$$

with $\zeta_{q=1} = \zeta_{q=\frac{2}{3}} = 1$ and $\zeta_{q=2} = \frac{2}{3}$ [19]. Finally, the q-tensor-scalar ratio from Eqs.(22) and (27) becomes

$$r_q = 16\frac{A_{T,q}^2}{A_{S,q}^2} = 16\frac{\epsilon_1^q}{\zeta_q} = \frac{16}{\zeta_q}\frac{qA^{2\alpha_0\alpha_3}\alpha_4^{1-q}}{6\beta_q^2}\left[\frac{2\,(1-f)}{q\,(\alpha_1+1)}\right]^2\phi^{-2\gamma}, \quad (28)$$

in the patch cosmological models.

From Eqs.(25) and (28) we can write the relation between the tensor-to-scalar ratio r_q and the spectral index n_s^q as

$$r_q(n_s^q) = \frac{8(1-f)}{\zeta_q(\alpha_1+1)}\frac{(1-n_s^q)}{\left[\frac{(1-f)}{\alpha_1+1}\left(3-\frac{2}{q}\right)-1\right]}. \quad (29)$$

Also, we can write the relation between the number of e-folds N and the tensor-to-scalar ratio r_q, from Eqs.(21) and (28) as

$$N = \alpha_8\left(\frac{16q\alpha_4^{1-q}}{6\zeta_q\beta_q^2}\left[\frac{2(1-f)}{q(\alpha_1+1)}\right]^2 \frac{1}{r_q} - \alpha_6\right). \quad (30)$$

In Fig.(1) we show the dependence of the tensor-scalar ratio on the spectral index, from Eq.(29). From left to right q=2 to corresponds (RS), 1 (GR) and 2/3 (GB), respectively. From Ref.[44], two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r, the tensor-scalar ratio, and n_s, the spectral index of fluctuations, defined at $k_0 = 0.002$ Mpc^{-1}. The five-year WMAP data places stronger limits on r (shown in blue) than three-year data (grey)[45]. In order to write down values that relate n_s and r, we used Eq.(29). Also we have used the value $f = 3/5$. From Eq.(30) and the line of RS for $q = 2$, we observed that for $f = \frac{3}{5}$, the curve $r = r(n_s)$ (see Fig. (1)) for WMAP 5-years enters the 95% confidence region for (RS) where the ratio $r_2 \simeq 0.33$, which corresponds to the number of e-folds, $N \simeq 47.2$. For $q = 1$ (GR), $r_1 \simeq 0.38$ corresponds to $N \simeq 27.3$. For $q = 2/3$ (GB), $r_{2/3} \simeq 0.52$ corresponds to $N \simeq 20.3$. From 68% confidence region for $q = 2$ (RS), $r_2 \simeq 0.28$, which corresponds to $N \simeq 57.3$. For $q = 1$ (GR), $r_1 \simeq 0.25$ corresponds to $N \simeq 41.7$, and for $q = 2/3$ (GB), $r_{2/3} \simeq 0.27$ corresponds to $N \simeq 39$.

From Eqs.(24) and (26) we can write the general relation in GB brane world for the tensor-to-scalar ratio r_q given by

$$r_q = 16\frac{A^2_{T,q}}{A^2_{S,q}} = \left[\frac{400\,\beta_q^2\,(f-1)^2\,A^{2\alpha_3(q-1)/q}\,\alpha_4^{(q-1)}}{3\,q^2\,(\alpha_1+1)^2\,\beta_1^4}\right]\,\phi_*^{-2\,\gamma}\,\frac{F^2_\beta(x_*)}{G^2_\beta(x_*)}, \tag{31}$$

where $x_* \equiv H_*/\mu$.

In Fig.(2) we show the dependence of the tensor-scalar ratio on the spectral index, from Eqs.(25) and (31). Here, we have taken two different values of the GB parameter $\beta^2_{2/3}$. In doing this, we have used values $f = 3/5$, $A = 10^{-3}\,m_p^{2/3}$, and $\beta = 10^{-3}$, respectively. Note that the Fig.(2), becomes different to the Fig.(1) for the case $q = 2/3$, when we have used the corrections given by Eq.(31) .

Numerically from Eq.(21), we observed that for the parameter $\beta^2_{2/3} = 10^{-4} m_p^{-2/3}$ the curve $r = r(n_s)$ (see Fig. (2)) for WMAP 5-years enters the 95% confidence region the ratio $r_{2/3} \simeq 0.41$, which corresponds to the number of e-folds, $N \simeq 26$. For $\beta^2_{2/3} = 10^{-5} m_p^{-2/3}$, $r_{2/3} \simeq 0.42$ corresponds to $N \simeq 25$. Note also that the curve-value $\beta_{2/3} = 10^{-4} m_p^{-2/3}$ does not agree with the one-dimensional marginalized constraint 68% confidence level on inflationary parameters r, this is due to its curve is obtained for a given values of A, β and f.

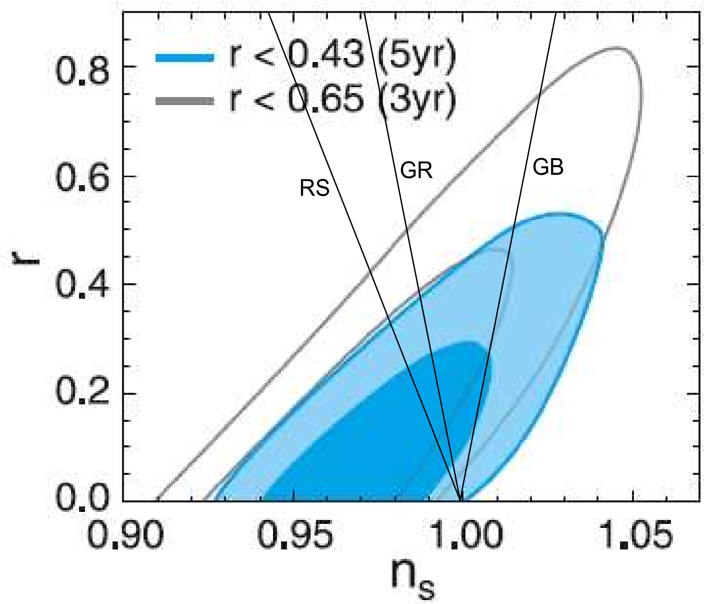

FIG. 1: The plot shows r versus n_s. Here, we have fixed the value $f = 3/5$. The five-year WMAP data places stronger limits on the tensor-scalar ratio (shown in blue) than three-year data (grey) [44]. The choices $q = 1, 2, 2/3$, corresponds to the General Relativity (GR), Randall Sundrum (RS) and Gauss-Bonnet (GB) regimes, respectively.

V. CONCLUSION AND FINAL REMARKS

In this work we have studied an intermediate inflationary universe model in which the gravitational effects are described by the Gauss-Bonnet Brane World Cosmology. We study this model by using the scheme of patch cosmology. In this approach the dynamics of the scale factor is governed by a modified Friedmann equation given by $H^2 = \beta_q^2 \rho^q$, where $q = 1$ represent GR theory, $q = 2$ describes high energy limit of brane world cosmology, and $q = 2/3$ corresponds to brane world cosmology with a Gauss-Bonnet correction in the bulk. We have described different cosmological models where the matter content is given by a single scalar field in presence of the power-law potential. By using the scalar potential (see Eq.(16)) and from the WMAP five year data, we have found constraints on the parameter A for a given values of β_q and f (see Eq.(23)). In particular, for $f = 2/3$ we obtained that

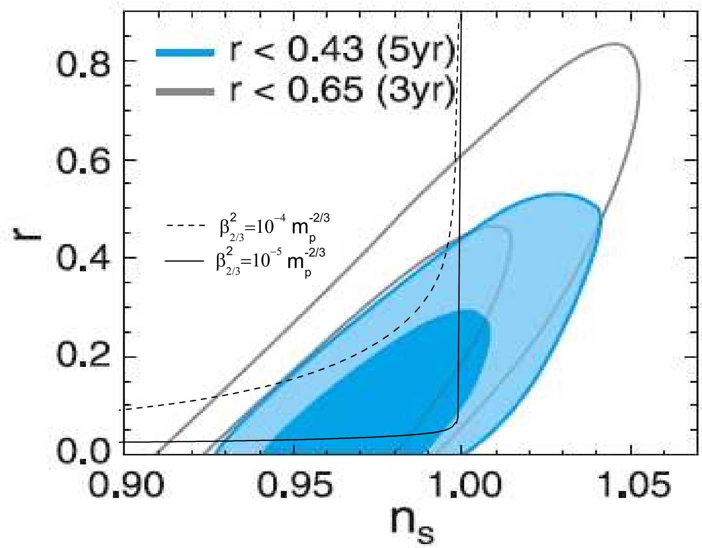

FIG. 2: The plot shows r versus n_s, for two different values of the GB parameter $\beta^2_{2/3}$. Here, we have fixed the values $f = 3/5$, $A = 10^{-3}\, m_p^{2/3}$ and $\beta = 10^{-3}$, respectively.

in GR ($q = 1$) we get $A \simeq 0.0019 m_p^{2/3}$. In RS ($q = 2$) for the value of $\beta_2^2 = 10^{-13} m_p^{-6}$, we have $A \simeq 0.0014 m_p^{2/3}$, and in the GB regime ($q = 2/3$) for $\beta^2_{2/3} = 10^{-3} m_p^{-2/3}$, we get $A \simeq 0.0046 m_p^{2/3}$. Here we have taken $N = 60$ and $\mathcal{P}_{\mathcal{R}} \simeq 2.4 \times 10^{-9}$. In order to bring some explicit results we have taken the constraint $r - n_s$ plane to first-order in the slow roll approximation. We noted that the parameter f, which lies in the range $1 > f > 0$, the model is well supported by the data as could be seen from Fig.(1).

In this paper, we have not addressed the phenomena of reheating and possible transition to the standard cosmology (see e.g., Refs.[46–48]). A possible calculation for the reheating temperature in the high-energy scenario would give new constrains on the parameters of the model. We hope to return to this point in the near future.

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